

# IUSEP Project

Do some or all of the following.

- 1 Prove the isoperimetric inequality. While a proof of convergence to a circle is too difficult, try to sketch some of the reasoning.
- 2 A standard torus in  $\mathbb{R}^3$  is any torus in the family of surfaces of revolution obtained by revolving the profile curve

$$\gamma(u) = (a + b \cos u, 0, b \sin u), \quad a \geq b > 0, \quad u \in [0, 2\pi]$$

about the  $z$ -axis (the vertical axis). For a standard torus  $\mathbb{T}$ :

- a) Find the principal curvatures  $\kappa_1, \kappa_2$ . Find the mean curvature  $H$  and Gauss curvature  $K_G$ .
- b) Compute the *Willmore energy*  $W(a/b) = \int_{\mathbb{T}} H^2 dA$ .
- c) If  $z = \frac{a}{b}$ , find  $z$  such that  $W(z)$  is a minimum. Standard tori with  $\frac{a}{b}$  given by this value are called *Willmore tori*.
  - Find out what you can about the Willmore conjecture. What is it, and has it been proved? Is there a version for other surfaces?