

IUSEP Project

Do some or all of the following.

- ➊ Prove the isoperimetric inequality. While a proof of convergence to a circle is too difficult, try to sketch some of the reasoning.
- ➋ A standard torus in \mathbb{R}^3 is any torus in the family of surfaces of revolution obtained by revolving the profile curve

$$\gamma(u) = (a + b \cos u, 0, b \sin u), \quad a \geq b > 0, \quad u \in [0, 2\pi]$$

about the z -axis (the vertical axis). For a standard torus \mathbb{T} :

- Find the principal curvatures κ_1, κ_2 . Find the mean curvature H and Gauss curvature K_G .
- Compute the *Willmore energy* $W(a/b) = \int_{\mathbb{T}} H^2 dA$.
- If $z = \frac{a}{b}$, find z such that $W(z)$ is a minimum. Standard tori with $\frac{a}{b}$ given by this value are called *Willmore tori*.
- Find out what you can about the Willmore conjecture. What is it, and has it been proved? Is there a version for other surfaces?