IUSEP suggested problems on optimal transport and applications

1) Density functional theory leads to an optimal transport problem where the two densities are the same and the cost is $c(x,y) = 1/|x-y|$. The aim of this project is to:
-Learn background about the density functional theory problem. (How does it arise in physics/chemisty? What is the main goal?)
-Understand how optimal transport arises in this context, in connection with the semi-classical limit?
-Understand some of the mathematics behind optimal transport with Coulomb cost $c(x,y) = 1/|x-y|$.
Some particular questions which might be of interest are:

-If the densities are supported on the real line (that is, in one dimension), can you work out the solution (this is similar to the one dimensional solution studied in the lecture, but the cost function here doesn't satisfy the same conditions)?

-If there are three (or, more generally, several), rather than two electrons, what is the mathematical analog of the optimal transport problem? Is it possible to have unique solutions here? In one dimension, can you work out the solution(s) by hand?

2) Hedonic pricing is a problem in economics where buyers and sellers match together according to their preferences to buy (respectively sell) different goods. Try to understand the mathematical structure of this problem (it can be formulated in terms of optimal transport maps). Can the resulting mathematical problem be given a “factories and mines” type of interpretation (hint, imagine that you have two, rather than one, type of factory, producing say iron and aluminum, and you want to build your mines in locations that minimize some total transport cost). Can the problem also be rewritten as a matching (that is, pure optimal transport) problem?