Project

Do some or all of the following.

1. Prove the 4 theorems on the last slide of my second lecture.

2. A standard torus in $\mathbb{R}^3$ is any torus in the family of surfaces of revolution obtained by revolving the profile curves

$$\gamma(u) = \left(a + b \cos \frac{u}{b}, 0, b \sin \frac{u}{b} \right), \ a \geq b, \ u \in [0, 2\pi),$$

about the z-axis (the vertical axis). For a standard torus $\mathcal{T}$:

   a) Find κ_1, κ_2, H, K_G.
   b) Compute the *Willmore energy* $W(a/b) = \int_{\mathcal{T}} H^2 dA$.
   c) The Willmore energy of a standard torus is $W(z)$ is a function of the single variable $z = a/b$. Find z such that $W(z)$ is a minimum. Standard tori with a/b given by this value are called Willmore tori.
   d) Find out what you can about the Willmore conjecture. Has the conjecture been proved for general tori? Is there a version for other surfaces?