IUSEP, Mathematical Finance, Project 1

In Section 1 of the lecture on mathematical finance, we studied a model with one stock (risky asset) and one bank account (risk-free asset). Such a risk-free asset may not always be available. In this project, we consider a model with two stocks, but no bank account. The initial values of the stocks are S_0 and Y_0. Their values at time 1 are either uS_0 and hY_0 with probability p, or dS_0 and lY_0 with probability $1-p$. For the project, you can consider the following different aspects of this model.

1. *No-arbitrage condition.* Find the no-arbitrage condition for this model and explain it in financial terms.

2. *Pricing a financial derivative.* Assume that the no-arbitrage condition that you found in 1. is satisfied and consider a financial derivative in addition to the two stocks. The financial derivative has payoff f_u if the values of the two stocks are uS_0 and hY_0. The financial derivative has payoff f_d if the values of the two stocks are dS_0 and lY_0.

   (a) Find formulae (by hand, not using MATLAB) for

      i. the units of the first stock held in the replicating portfolio,

      ii. the units of the second stock held in the replicating portfolio,

      iii. the fair price of the financial derivative in this model.

   (b) Write a MATLAB function that returns the price of the financial derivative in this model.

   (c) Consider a derivative with payoff $f_u = 2uS_0$ and $f_d = 0$. The parameters of the stocks are given by $S_0 = \$20$, $Y_0 = \$25$, $u = 1.4$, $d = 0.8$, $h = 1.5$ and with different values for l. Create a plot of the derivative price as a function of $l = 0, 0.01, 0.02, \ldots, 0.6$.

   (d) For a derivative with payoff $f_u = 2uS_0$ and $f_d = 2dS_0$, plot of the derivative price as a function of $l = 0, 0.01, 0.02, \ldots, 0.6$. The parameters of the stocks are the same as in (c).

   (e) Explain the different shapes of the curves in (c) and (d).

3. *Replicating strategy.* We now focus on the replicating portfolio.

   (a) Using 2(a) i. and ii., write a MATLAB function that returns the replicating strategy (Δ_1, Δ_2) for a derivative $f = (f_u, f_d)$.

   (b) Use your previous MATLAB function to write a MATLAB function that returns the replicating strategy (Δ_1, Δ_2) for a call option on the first stock with strike price K.

   (c) Consider a call option on the first stock. The parameters of the stocks are given by $S_0 = \$20$, $Y_0 = \$25$, $u = 1.4$, $d = 0.8$, $h = 1.5$ and $l = 0.5$. Create a plot of the replicating portfolio (Δ_1, Δ_2) as a function of $K = 0, 0.1, 0.2, \ldots, 40$.

(d) Explain why the curves in your plot in (c) are constant for $K \geq 30$.

(e) Explain the values of (Δ_1, Δ_2) at $K = 0$ in your plot in (c).

4. *Multiperiod model.* Expand the model to n periods. What will be the fair price of the financial derivative at time zero? Can this price be considered as the discounted expectation of the derivative payoff under a risk-neutral probability measure?